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ON THE ATTENUATION OF GUIDED WAVES IN THE LIMIT OF HIGH FREQUENCIES

by

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Abstract

The conventional formulas for the attenuation of waves due to the wall losses in uniform waveguides are based on the two assumptions that the wall currents are the same as the loss-free currents and that the surface resistance of the highly conductive walls is isotropic. In the limit of high frequencies the former assumption remains valid whereas the latter assumption breaks down. As the frequency is increased the surface resistance becomes anisotropic in the sense that it assumes different values depending on whether the wall current is longitudinal or transverse. In this paper new attenuation formulas are derived, which take into account the high-frequency anisotropy of the surface resistance and hence yield accurate results for all frequencies.

Introduction

According to the conventional theory of the attenuation of electromagnetic waves in uniform waveguides with slightly lossy walls, the general dependence of the attenuation parameters on the frequency of oscillation is as follows:* For each E-mode the attenuation parameter α_E is large near cut-off, decreases rather rapidly to a minimum, and then rises steadily as the frequency is increased. In the limit of high frequencies α_E depends on frequency as $\omega^{1/2}$. For each H-mode the attenuation parameter α_H is large near cut-off and rapidly decreases as the frequency is

*Southworth, G.C., Bell System Tech. J., 15, 284 (1936); Carson, J.R., S.P. Mead, S. A. Schelkunoff, Bell System Tech. J., 15, 310 (1936).

increased. As the frequency is increased still further the contribution to α_H of the longitudinal wall currents behaves as $\omega^{1/2}$ whereas the contribution of the transverse wall currents behaves as $\omega^{-3/2}$. Plots of α_E and α_H versus ω have the same general shape (lazy U) except in the case of an anomalous H-mode which induces only transverse wall currents and hence has an attenuation parameter that decreases monotonically with frequency.*

Upon examination of these results one may rightly suspect that the conventional theory in the limit of high frequencies is free from objection only in the case of an anomalous H-mode because for all other modes it yields attenuation parameters that increase indefinitely with frequency. Actually α_E and α_H should eventually decrease with frequency rather than increase without limit. Indeed, it has been shown by Pannenborg** that the attenuation parameter of an E_{01}-mode in a circular guide is large near cut-off, falls to a minimum, then rises to a maximum, and finally decreases to zero as the frequency is increased from its cut-off value. The fact that the attenuation reaches a maximum beyond which it monotonically tapers to zero shows that at high frequencies the conventional theory breaks down, at least in the case of this mode. The question of whether this behavior is a property of all non-anomalous modes in uniform waveguides of arbitrary cross-section has not been settled, and it is the purpose of this paper to do so. The method we shall use in deriving the new formulas for α_E and α_H that are valid for all frequencies of oscillation is the same as the conventional method except for the fact

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*S. E. Miller, Bell System Tech. J. 33, 1209 (1954).

**A. E. Pannenborg, Report No. 2197, April 1948, Philips Research Laboratories, Eindhoven.

that in computing the ohmic wall losses the anisotropic surface resistance is used instead of the conventional isotropic surface resistance. To find the anisotropic surface resistance the waveguide modes are expressed as TEM-waves traveling down the structure in zigzag paths by multiple reflections from the walls* where the absorption of these elementary waves is assumed to be the same as the absorption at a plane conducting surface.**

Apparent Anisotropy of Surface Resistance

We consider the ohmic losses that occur when a plane TEM wave which is polarized parallel or perpendicular to the plane of incidence falls on a plane conducting surface (Fig. 1). We assume that the conductivity of the conducting surface is high and the half-space above it is a vacuum. The Poynting vector of the incident wave is

$$\underline{S}^{inc} = \frac{1}{2}\,\underline{E}_o \times \underline{H}^*_o = \frac{1}{2}\,\eta\, H_o^2\, \underline{n}_o \tag{1}$$

where $\underline{E}_o$ and $\underline{H}_o$ are respectively the electric and magnetic field intensity vectors of the incident wave, $\underline{n}_o$ is the unit wave-normal, μ_o and ϵ_o are respectively the permeability and the dielectric constant of the vacuum, and $\eta = \sqrt{\mu_o/\epsilon_o}$. The energy incident per second on a unit area of the conducting surface is $\underline{n} \cdot \underline{S}^{inc}$ or $\frac{1}{2}\,\eta\, H_o^2 \cos\Theta$ where Θ is the angle of incidence with respect to the unit normal $\underline{n}$ of the surface. Hence the energy dissipated per second per unit area of the conducting surface is

$$w_{||} = \frac{1}{2}\,\eta\, H_o^2\, A_{||} \cos\Theta \tag{2}$$

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*L. Brillouin, Revue Générale de l'Electricité, 40, 227 (1936).
**W.E. Lamb, Jr., Phys. Rev. 70 308 (1946).

for parallel propagation, and

$$w_{\perp} = \frac{1}{2}\,\eta\, H_o^2\, A_{\perp} \cos\Theta \tag{3}$$

for the case of perpendicular polarization. The absorption coefficients $A_{||}$ and $A_{\perp}$ are given by*

$$A_{||} = \frac{4p\cos\Theta}{2\cos^2\Theta + 2p\cos\Theta + p^2}\,, \qquad A_{\perp} = 2p\cos\Theta \tag{4}$$

where

$$p = \frac{2}{\eta}R \qquad \text{and} \qquad R = \sqrt{\frac{\omega\,\mu_M}{2\sigma_M}}\,.$$

Here R denotes the conventional surface resistance, μ_M and σ_M the permeability and conductivity of the metal, and as before, $\eta = \sqrt{\mu_o/\epsilon_o}$. It follows from equations (2), (3), and (4) that the power dissipated per unit area of the conducting surface is

$$w_{||} = 2R\,H_o^2\,\frac{1}{1 + \dfrac{p}{\cos\Theta} + \dfrac{p^2}{2\cos^2\Theta}} \tag{5}$$

for parallel polarization, and

$$w_{\perp} = 2R\,H_0^2\cos^2\Theta \tag{6}$$

for perpendicular polarization.

In conventional waveguide theory incidental wall losses are given by $\frac{1}{2}\,\underline{K}\cdot\underline{K}^*R$ per unit area of wall, where $\underline{K}$ is the surface current density that would exist if the wall were perfectly conducting. If $\underline{H}$ is the resultant magnetic vector at the lossless surface, then $\underline{K} = \underline{n} \times \underline{H}$ or

*See, for example, J. A. Stratton, Electromagnetic Theory, New York/London 1941, pp.507-508, equations (87) and (88)

$K_z = 2H_o$ for parallel polarization and $K_x = 2H_o \cos\Theta$ for perpendicular polarization. In terms of the longitudinal component K_z and the transverse component K_x of the surface current $\underline{K}$, expressions (5) and (6) are

$$w_{||} = \frac{1}{2} K_z^2 R \frac{1}{1 + \frac{p}{\cos\Theta} + \frac{p^2}{2\cos^2\Theta}} \tag{7}$$

$$w_{\perp} = \frac{1}{2} K_x^2 R \quad . \tag{8}$$

Thus we see that the expression for $w_{\perp}$ is the same as that of the conventional theory whereas the expression for $w_{||}$ differs from that of the conventional theory by the multiplicative factor $(1 + p/\cos\Theta + p^2/2\cos^2\Theta)^{-1}$.

With respect to the z-direction the case of parallel polarization is equivalent to an E-wave that induces purely longitudinal currents, and the case of perpendicular polarization is equivalent to an H-wave that induces purely transverse currents. The attenuation constant may be calculated in the conventional manner provided the surface resistance of the conductor is taken to be

$$R_{\ell} = \frac{R}{1 + \frac{p}{\cos\Theta} + \frac{p^2}{2\cos^2\Theta}} \tag{9}$$

for longitudinal currents, and

$$R_t = R \tag{10}$$

for transverse currents. In this sense the apparent surface resistance is anisotropic.

Surface Resistance of Uniform Guides

The longitudinal and transverse (circumferential) currents along the walls of a uniform guide may be computed under the assumption that the walls are losses. E-waves produce purely longitudinal currents. Non-anomalous H-waves give rise to both longitudinal and transverse currents. The angle of incidence Θ may be found by decomposing the modes into TEM waves traveling down the guide in a zigzag course by multiple reflections from the walls (Fig. 2). For any mode

$$\sin\Theta = \sqrt{1-(\gamma/k)^2} \tag{11}$$

where γ denotes the cut-off wave number, and k the free-space wave number. Since $p = 2R/\eta$, it follows from equation (11) that

$$\frac{p}{\cos\Theta} = \frac{2}{\eta}\,\frac{k}{\gamma}\,R = \frac{2\omega\epsilon_o R}{\gamma} \; . \tag{12}$$

Substituting expression (12) into equation (9) we see that

$$R_{\ell} = \frac{R}{1+\frac{2\omega\epsilon_o R}{\gamma}+\frac{1}{2}\left(\frac{2\omega\epsilon_o R}{\gamma}\right)^2} \; , \tag{13}$$

and from equation (10) we see that

$$R_t = R \; . \tag{14}$$

New Attenuation Formulas

In the conventional theory the attenuation parameters are given by*

$$\alpha_E = \frac{1}{2}\,R\,\frac{\omega\epsilon_o}{h_E}\,\frac{\oint\left(\frac{\partial\phi}{\partial n}\right)^2 d\ell}{\gamma_E^2\int\phi^2\,dA} \tag{15}$$

*See, for example, F. E. Borgnis and C. H. Papas, "Electromagnetic Waveguides and Cavity Resonators", an article in the Encyclopedia of Physics, Springer-Verlag, Berlin/Göttingen/Heidelberg, Vol.16, 1958, p.320 .

for E-modes, and by

$$\alpha_H = \frac{1}{2} R \frac{h_H}{\omega\mu_o} \frac{\oint (\frac{\partial\psi}{\partial\ell})^2 \, d\ell}{\gamma_H^2 \int \psi^2 \, dA} + \frac{1}{2} R \frac{\gamma_H^2}{\omega\mu_o h_H} \frac{\oint \psi^2 \, d\ell}{\int \psi^2 \, dA} \tag{16}$$

for H-modes. In these formulas ϕ and ψ are the eigenfunctions of the waveguide with corresponding eigenvalues γ_E and γ_H, and propagation constants $h_E = (k^2 - \gamma_E^2)^{1/2}$ and $h_H = (k^2 - \gamma_H^2)^{1/2}$. Also $\int dA$ and $\oint d\ell$ denote integrations respectively over a cross-section of the guide and along its periphery. E-modes generate only longitudinal currents and hence in the formula for α_E we must replace R by R_ℓ. The first term of the formula for α_H is associated with purely longitudinal currents, the second with purely transverse currents. Hence in the first term R must be replaced by R_ℓ.

Thus the formulas for α_E and α_H become

$$\alpha_E = \frac{1}{2} R \frac{\omega\epsilon_o}{h_E} \frac{\oint (\frac{\partial\phi}{\partial n})^2 \, d\ell}{\gamma_E^2 \int \phi^2 \, dA} \left[\frac{1}{1 + \frac{2\omega\epsilon_o R}{\gamma} + \frac{1}{2} (\frac{2\omega\epsilon_o R}{\gamma})^2} \right] \tag{17}$$

$$\alpha_H = \frac{1}{2} R \frac{h_H}{\omega\mu} \frac{\oint (\frac{\partial\psi}{\partial\ell})^2 \, d\ell}{\gamma_H^2 \int \psi^2 \, dA} \left[\frac{1}{1 + \frac{2\omega\epsilon_o}{\gamma} + \frac{1}{2} (\frac{2\omega\epsilon_o R}{\gamma})^2} \right] + \frac{1}{2} R \frac{\gamma_H^2}{\omega\mu h_H} \frac{\oint \psi^2 \, d\ell}{\int \psi^2 \, dA} . \tag{18}$$

To place in evidence the dependence of these expressions on the frequency, we let

$$C_E = \frac{\epsilon_o}{\gamma_E} \sqrt{\frac{2\mu_M}{\sigma_M}} , \qquad C_H = \frac{\epsilon_o}{\gamma_H} \sqrt{\frac{2\mu_M}{\sigma_M}} ,$$

$$M = \frac{1}{2}\sqrt{\frac{\mu_M}{2\sigma_M}}\ \frac{\epsilon_0}{\gamma_E^2}\ \frac{\oint \left(\frac{\partial \phi}{\partial n}\right)^2 d\ell}{\int \phi^2\, dA}\ , \qquad N = \frac{1}{2}\sqrt{\frac{\mu_M}{2\sigma_M}}\ \frac{1}{\mu_0\, \gamma_H^2}\ \frac{\oint \left(\frac{\partial \psi}{\partial \ell}\right)^2 d\ell}{\int \psi^2\, dA}\ ,$$

and

$$Q = \frac{1}{2}\sqrt{\frac{\mu_M}{2\sigma_M}}\ \frac{\gamma_H^2}{\mu_0}\ \frac{\oint \psi^2\, d\ell}{\int \psi^2\, dA}\ .$$

We see that C, M, N, and Q are mode-dependent but frequency-independent. With the use of the constants formulas (17) and (18) may be written as

$$\alpha_E = M\ \frac{\omega^{3/2}}{(\omega^2\, \mu_0\epsilon_0 - \gamma_E^2)^{1/2}}\ \left[\frac{1}{1 + C_E\omega^{3/2} + \frac{1}{2}\, C_E^2\, \omega^3}\right] \tag{19}$$

$$\alpha_H = N\, \frac{(\omega^2\, \mu_0\epsilon_0 - \gamma_H^2)^{1/2}}{\omega^{1/2}}\ \left[\frac{1}{1 + C_H\omega^{3/2} + \frac{1}{2}\, C_H^2\, \omega^3}\right] + $$

$$+\ Q\ \frac{1}{\omega^{1/2}}\ \frac{1}{(\omega^2\, \mu_0\epsilon_0 - \gamma_H^2)^{1/2}}\ . \tag{20}$$

These are the new attenuation formulas and they differ from those of the conventional theory by the factors in square brackets, that is, in the conventional theory the square-bracketed factors are equal to unity.

In the conventional theory it is assumed that $R_\ell = R_t = R$. Indeed this is a good approximation at sufficiently low frequencies where the terms $2\omega\epsilon_0 R/\gamma$ and $(1/2)(2\omega\epsilon_0 R/\gamma)^2$ are negligible with respect to unity. However, in the limit of high frequencies account must be taken of these terms.

Clearly, at high frequencies α_E and α_H (non-anomalous) depend on frequency as $\omega^{1/2}/(1 + C\omega^{3/2} + \frac{1}{2}\, C^2\, \omega^3)$. This function has a maximum at a frequency several magnitudes greater than the cut-off frequency and then tapers to zero, the limiting frequency dependence being $\omega^{-5/2}$.

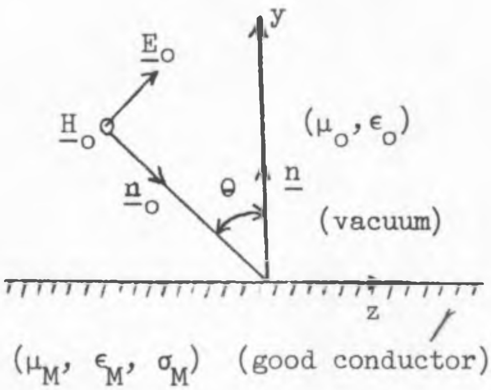

Fig. 1a. Incident TEM wave polarized parallel to plane of incidence and traveling in direction $\underline{n}_o$.

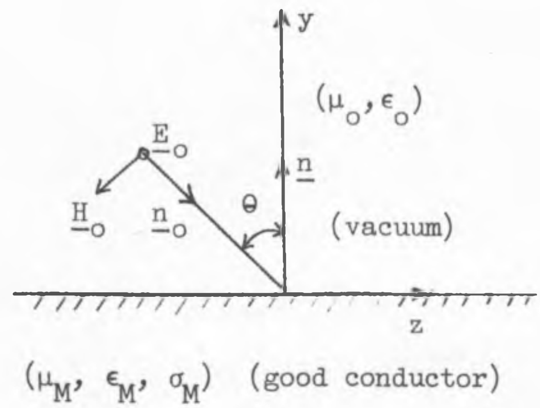

Fig. 1b. Incident TEM wave polarized perpendicular to plane of incidence and traveling in direction $\underline{n}_o$.

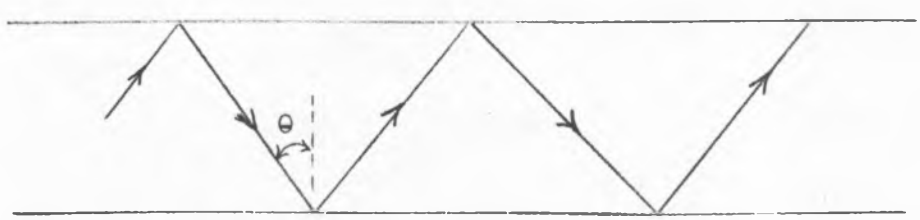

Fig. 2. Zigzag course of TEM wave traveling down uniform waveguide.

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